

# The Contribution of the Quark Condensate to the $\pi$ N Sigma Term

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## Abstract

There has been a discrepancy between values of the pion-nucleon sigma term extracted by two different methods for many years. Analysis of recent high precision pion-nucleon data has widened the gap between the two determinations. We argue that the two extractions correspond to different definitions and that the difference between them can be understood and calculated.

The sigma term is directly related to explicit chiral symmetry breaking in the nucleon. The conventional view has been that its value can be obtained by either an extrapolation of the isospin zero pion-nucleon scattering amplitude to a defined subthreshold point or by comparing masses of members of the baryon octet. There has long been a problem that the two determinations do not result in the same value. We argue that they *should not have the same value* and that the difference is about what one should expect.

The value extracted from the comparison of masses ( $\Sigma_M$ ) is commonly thought to be around 35 MeV while the value of the sigma term extracted from pion-nucleon scattering ( $\Sigma_S$ ) was given by Koch as  $64 \pm 8$  MeV [1] using dispersion analysis and the Karlsruhe-Helsinki phase shifts. While this difference was disturbing, there were theoretical corrections to be made and the pion-nucleon data at that time were not of high quality, especially at low energy.

It was suggested that the difference could be interpreted as evidence for a large component of strangeness in the nucleon [3, 2]. Estimates of the strangeness content from the strange meson cloud give around 7.6% [4]. From neutrino induced reactions the strangeness content has been reported as 6.4 % [5] and 9.9 % [6] (although corrections to these numbers may be significant [7]), making the 20% required for this explanation of the discrepancy questionable.

The  $\pi$ N data base has been improved recently (see Ref. [8] for an analysis). New sigma-term extractions with this data, however, did not reduce the discrepancy, but increased it.

Kaufmann and Hite[9] used interior and fixed  $t$  dispersion relations to map out the subthreshold amplitude. They obtained a value of  $\Sigma_S = 88 \pm 15$  MeV. Recently Olsson and

Kaufmann[10], using Olsson's sum rule, found  $\Sigma_S = 80 - 90$  MeV. Pavan[11] reported on a study using the newest VPI/GWU [12] analysis of data which found  $\Sigma_S = 79 \pm 7$  MeV.

In order to find the value at the soft pion point, a correction is needed. Modern estimates give a 15 MeV reduction [15, 16, 13, 14]. It has been suggested that, based on PCAC, this correction is likely to be too large[17]. Lattice calculations[18] give a decrease of 6.6 MeV. The value of the sigma term extracted from pion-nucleon scattering would appear to be  $85 - 15 = 70 \pm 7$  MeV, or perhaps somewhat greater.

The evaluation of the sigma commutator with the use of an effective Lagrangian [19] shows it to be proportional to the nucleonic expectation value of  $m_u \bar{u}u + m_d \bar{d}d$  [ $\frac{1}{2}(m_d + m_u)(\bar{u}u + \bar{d}d)$  for charged pions] in the nucleon. In the following we neglect the quark mass difference (i.e. we take  $m_u = m_d = m_q$ ) and define  $\bar{q}q = \frac{1}{2}(\bar{u}u + \bar{d}d)$ .

In a theory with spontaneous symmetry breaking one cannot assume the commonly used definition

$$\Sigma = 2m_q \int d^3x \langle N | \bar{q}q(x) | N \rangle \quad (1)$$

where the state  $|N\rangle$  is interpreted as a system which is localized in space within a nucleonic volume. Since outside this region  $\langle N | \bar{q}q(x) | N \rangle = \langle 0 | \bar{q}q(x) | 0 \rangle$  is a constant, the integral diverges. To obtain a finite result we follow several authors [20, 21, 22, 23] to define

$$\Sigma_S = 2m_q \int d^3x [\langle N | \bar{q}q(x) | N \rangle - \langle 0 | \bar{q}q(x) | 0 \rangle]. \quad (2)$$

To estimate the size of the vacuum correction within the nucleon, we consider the model of the nucleon as a constant density over a sphere of radius R and volume V and rewrite the last equation as

$$\Sigma_S = 2m_q \int_V d^3x \langle N | \bar{q}q(x) | N \rangle - 2m_q V \langle 0 | \bar{q}q | 0 \rangle. \quad (3)$$

The nucleon averaged value is defined as

$$\overline{\langle N | \bar{q}q | N \rangle} = \frac{1}{V} \int_V d^3x \langle N | \bar{q}q(x) | N \rangle. \quad (4)$$

The GOR [24] relation allows us to express

$$2m_q \langle 0 | \bar{q}q | 0 \rangle = -m_\pi^2 f_\pi^2 = -21.7 \text{ MeV/fm}^3 \quad (5)$$

where  $f_\pi$  is the pion decay constant (92.4 MeV).

With the definition

$$\delta = -2m_q V \langle 0 | \bar{q}q | 0 \rangle = V m_\pi^2 f_\pi^2, \quad (6)$$

we can write Eq. 3 as

$$\Sigma_S = 2m_q V \overline{\langle N | \bar{q}q | N \rangle} + \delta. \quad (7)$$

While the second term may seem unfamiliar, we point out that the sigma term obtained from the linear sigma model [25] depends only on the *vacuum* expectation value of the sigma field (often identified with  $\bar{q}q$ ).

As an illustration, we consider a “bubble” model for the  $\pi^0 p$  (i.e. isospin even) scattering amplitude in which the pion wave function satisfies a Klein-Gordon (KG) equation. Outside of the bubble of radius  $R$  the pion has momentum, mass, and energy given by  $k$ ,  $m_\pi$ , and  $\omega = \sqrt{k^2 + m_\pi^2}$ . Within the bubble, the interaction of the pion with the partons is represented by a constant, energy-independent potential,  $v$ . The restoration of chiral symmetry is modeled by setting the pion’s effective mass to zero in this region. The KG equation inside the sphere yields for the wave number in the interior,  $k_0$

$$k_0^2 + 0 = (\omega - v)^2. \quad (8)$$

To evaluate  $v$ , we impose the requirement that the scattering amplitude vanish at threshold (suggested by the data and chiral symmetry), which demands that the inner and outer momenta be equal at this point ( $k_0 = k = 0$ ) and establishes the value of  $v$  as  $m_\pi$  thus leading to

$$k_0^2 = (\omega - m_\pi)^2. \quad (9)$$

The scattering amplitude corresponding to the lowest order solution of the KG equation (good to a few percent when compared to the exact solution over the energy range considered here) is given by

$$f_0(k) = \frac{1}{3}(k_0^2 - k^2)R^3 = \frac{2}{3}m_\pi(m_\pi - \omega)R^3 \rightarrow \frac{2}{3}m_\pi^2 R^3 \quad (10)$$

where the arrow indicates evaluation at  $\omega = 0$  (a very good approximation to its value at the Cheng-Dashen point). Agreement with the low energy amplitude[8] is good, if  $R$  is taken in the range 0.72–0.77 fm, depending on the details of the fit.

One can consider the two effects (pion mass zero and the potential  $v = m_\pi$ ) as generating separate amplitudes (equal at  $\omega = 0$ ) which add to give the result of Eq. 10. This simple model indicates that the vacuum subtraction term and the parton scattering term should give equal contributions to the sigma term. With the factor  $4\pi f_\pi^2$  and the GOR relation,  $\delta$  is reproduced by the amplitude arising from the effect of the zero mass of the pion alone.

We now consider the determination of the sigma term from baryonic mass differences. From Gasser[26], to lowest order in the quark mass, we can write

$$M_n = M_0 + m_q(P_u + P_d) + m_s P_s \quad (11)$$

$$M_{\Sigma^0} = M_0 + m_q(P_d + P_s) + m_s P_u \quad (12)$$

$$M_{\Xi^0} = M_0 + m_q(P_u + P_s) + m_s P_d \quad (13)$$

where  $P_x = V \overline{\langle n | \bar{x} x | n \rangle}$  and  $|n\rangle$  is the neutron state vector.

$M_0$  represents the mass of the baryon in the absence of explicit chiral symmetry breaking by the light quark masses. Neglecting the strangeness content of the nucleon and taking the appropriate combination of the masses to eliminate  $M_0$ , Eqs. 11-13 are solved for the combination  $P_u + P_d$ . Applying the Feynman-Hellmann theorem

$$\Sigma_{\pi N} = m_q \frac{dM_N}{dm_q} \quad (14)$$

to Eq. 11 yields [27]

$$\begin{aligned} \Sigma_M &= m_q(P_u + P_d) = 2m_q V \overline{\langle N | \bar{q} q | N \rangle} = \\ &\frac{m_q}{m_s - m_q} (M_{\Xi^0} + M_{\Sigma^0} - 2M_n) = 26.3 \text{ MeV}, \end{aligned} \quad (15)$$

which corresponds to the first term in Eq. 7. The numerical value results from an assumed quark mass ratio of  $m_s/m_q = 25$ . Cheng and Li [19] (see Eq. 5.257) derive a similar result,

$$\Sigma_M = \frac{3m_q}{m_q - m_s} (M_\Lambda - M_{\Xi^0}) = 25.9 \text{ MeV}. \quad (16)$$

No shift of the masses for the vacuum energy density was considered, but if it were it could simply be included in  $M_0$  and would no longer appear in the final calculation. Gasser calculated[26] a correction from higher orders in the quark mass of about 10 MeV (although see Ref. [28]). The final value from the mass analysis is often quoted as

$$\Sigma_M = 35 \pm 5 \text{ MeV}. \quad (17)$$

A comparison of Eqs. 7 and 15 yields

$$\Sigma_S = \Sigma_M + \delta. \quad (18)$$

To calculate the expected value of  $\delta$ , the volume of the confinement region of the nucleon needs to be known. One estimate of this radius can be obtained from form factors for pion coupling. For a monopole form factor, a mass of 800 MeV to 1 GeV was found [29]. The volumes corresponding to these values lead to a vacuum contribution in the range  $\delta = 22.7$  to 43.2 MeV. From a lattice calculation of the distribution of valence quarks [30] the equivalent square radius of 0.65 fm can be obtained, leading to  $\delta = 25.0$  MeV. Using the values of  $R$  found in the “bubble” model above we find  $\delta$  in the range 33.8-41.5 MeV.

Much the same physics appears in the bag model where it is supposed that there is an energy density,  $B$ , inside the bag which exerts a pressure which is normally treated as a parameter to be fit in the process of matching the model to the data. The exclusion of the

Authors	B(MeV/fm <sup>3</sup> )
Chodos et al.[34]	27.0
Chodos and Thorn[35]	32.1
Barnhill et al. [36]	39.5
DeGrand et al. [37]	57.6, 31.8 (1)
Chanowitz and Sharpe [38]	27- 66
Vasconcellos et al. [39]	20 (1)

Table 1: Bag constants obtained by various groups. The notation (1) indicates that a large quark mass was used.

physical vacuum energy density from the cavity provides the same effect as an energy density within the bag. In theories in which the bag constant is calculated from such considerations [32, 31] it can be written as

$$B = B^0 + B^{xSB} \quad (19)$$

where  $B^{xSB}$  is given by Eq. 5. The attempt to identify the vacuum energy density with the bag constant has a long history (see e.g. [31, 33, 32]). The difficulty in making a credible identification has been that the contribution of the gluons appears to be much larger than the empirical bag constants (Table I) whereas they are of the same order as the energy density due to the quark condensate. Whatever the resolution of this problem, there seems to be general agreement as to the inclusion of the quark condensate term in the vacuum energy density.

In bag models the mass of the nucleon is given by

$$M_N = \langle N | H_{\text{parton}} | N \rangle + BV. \quad (20)$$

A number of calculations have been made based on this formula [40, 41, 42, 4] where the sigma term was obtained from the first term representing the valence and sea quarks. The contributions from these two parton scattering terms were found to be roughly equal and give a contribution to the sigma term of 35 to 45 MeV. Applying the Feynman-Hellmann theorem to the second term, as well as the first, the additional  $\delta$  contribution arises naturally in these models.

Note that spontaneous symmetry breaking (non-zero  $\langle 0 | \bar{q}q | 0 \rangle$ ), explicit symmetry breaking ( $m_q \neq 0$ ) and a finite size nucleon ( $V \neq 0$ ) are all needed for this correction. Any theory which does not include all three will find zero for  $\delta$ . Thus, a non-zero value of the condensate is required and the difference in the two determinations of the sigma term can be taken as an indication of its existence.

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## References

- [1] R. Koch, Z. Phys. C15, 161(1982)
- [2] M. Sainio,  $\pi$ N Newsletter **10**, p. 13(1995)
- [3] J. F. Donoghue and C. R. Nappi, Phys. Lett. 168B, 105(1986)
- [4] V. E. Lyubovitskij, T. Gutsche, A. Faessler and E. G. Drukarev, Phys. Rev. D 63, 054026(2001)
- [5] W. H. Smith *et al.*, Nucl. Phys. B S31 262(1993)
- [6] A. O. Bazarko *et al.*, Z. Phys. C65, 189(1995)
- [7] C. Boros, J. T. Londergan and A. W. Thomas, Phys. Rev. D58, 114030(1998)
- [8] W. R. Gibbs, Li Ai and W. B. Kaufmann, Phys. Rev. C 57, 784(1998)
- [9] W. B. Kaufmann and G. E. Hite, Phys. Rev. C60, 055204(1999)
- [10] M. G. Olsson and W. B. Kaufmann, Pion-nucleon Newsletter No. 16 (2002)
- [11] M. M. Pavan, R. A. Arndt, I. I. Strakovsky and R. L. Workman, Pion-nucleon Newsletter No. 16 (2002)
- [12] VPI/GWU phase-shift analysis, available on the Internet at <http://gwdac.phys.gwu.edu/>
- [13] T. Becher and H. Leutwyler, Eur. Phys. J. C9, 643(1999)
- [14] L. Brown, W. J. Pardee, and R. D. Peccei, Phys. Rev. **D4**, 2801(1971)
- [15] V. Bernard, N. Kaiser and U.-G. Meissner, Z. Phys. C 60, 111(1993)
- [16] J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. B 253, 260(1991)
- [17] S. A. Coon and M. D. Scadron, J. Phys. G: Nucl. Part. Phys. 18, 1923(1992)

- [18] S. J. Dong, J.-F Lagaë and K. F. Liu, Phys. Rev. D54, 5496(1996)
- [19] Pa-Pei Cheng and Ling-Fong Li, “Gauge Theory of Elementary Particle Physics”, Clarendon Press 1984
- [20] T. D. Cohen, R. J. Furnstahl and D. K Griegel, Phys. Rev. C45,1881(1992); Phys. Rev. Lett. 67, 961(1991)
- [21] A. Gammal and T. Frederico, Phys. Rev. C57, 2830(1998)
- [22] M. Ericson, Nucl. Phys. A577, 147c(1994)
- [23] J-L. Ballot, M. Ericson and M. R. Robilotta, Phys. Rev. C61, 5202(2000)
- [24] M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. 175, 2195(1968)
- [25] V. Koch, Int. Jour. Mod. Phys. E Nucl. Phys. 6, 203(1997); M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705(1960)
- [26] J. Gasser, Pion-nucleon workshop 1984, p. 266 Los Alamos Report LA-11184-C, Ann. Phys. 136, 62(1981); T. P. Cheng, Phys. Rev. D13, 2161(1976)
- [27] J. Gasser 1981 op. cit. See eq. 12.3
- [28] I. Jameson, A. A. Rawlinson and A. W. Thomas, Aust. J. Phys. 47, 45(1994)
- [29] S. A. Coon and M. D. Scadron, Phys. Rev. **C42**, 2256(1990); Phys. Rev. **C23**, 1150(1981)
- [30] M. Lissia, M.-C. Chu, J. W. Negele and J. M. Grandy, Nucl. Phys. A 555, 272(1993)
- [31] E. V. Shuryak, Phys. Lett. 79B, 135(1978)
- [32] V. Gogohia and H. Toki, Phys. Rev. D 61, 036006(2000)
- [33] V. Gogohia and Gy. Kluge, Phys. Rev. D 62, 076008(2000)
- [34] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. D10, 2599(1974)
- [35] A. Chodos and Charles B. Thorn, Phys. Rev. D12, 2733(1975)
- [36] M. V. Barnhill, W. K. Cheng and A. Halprin, Phys. Rev. D20, 727(1979)
- [37] T. DeGrand, R. L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D12, 2060(1975)

- [38] M. S. Chanowitz and S. Sharpe, Nucl. Phys. B 222, 211(1983)
- [39] C. A. Z. Vasconcellos, H. T. Coelho, F. G. Pilotto, B. E. J. Bodmann, M. Dillig and M. Razeira, Eur. Phys. J. C 4, 115(1998)
- [40] M. C. Birse and J. A. McGovern, Phys. Lett. B 292, 242(1992)
- [41] R. L. Jaffe, Phys. Rev. D 21, 3215(1980)
- [42] I. Jameson, A. W. Thomas and G. Chanfray, J. Phys. G: Nucl. Par. Phys. 18, L159(1992)